

Written Methods For Addition, Subtraction, Multiplication and Division.



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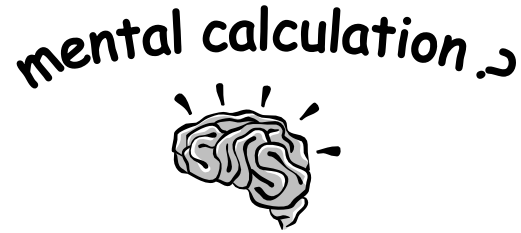
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“They didn’t do it like that in my day!”

Do your children ask for help with their maths homework and start talking in a foreign language, using words like ‘partitioning’, ‘chunking’, ‘grid multiplication’.....?

If so, you may feel the need for some translation.

Which is more important:



or



This will depend on the numbers involved and the individual child.

When faced with a calculation, no matter how large or difficult the numbers may appear to be, all children should ask themselves:

Can I do this in my head?

If I can't do it wholly in my head, what do I need to write down in order to help me calculate the answer?

Do I know the approximate size of the answer?

Will the written method I know be helpful?



When do children need to start recording?

ADDITION

Using an informal method by counting on in multiples of 10 with a number line

$$\begin{array}{r} T0 + T0 \\ 86 + 57 \end{array}$$

Why use a number line?

It helps me to show on paper what is going on in my head

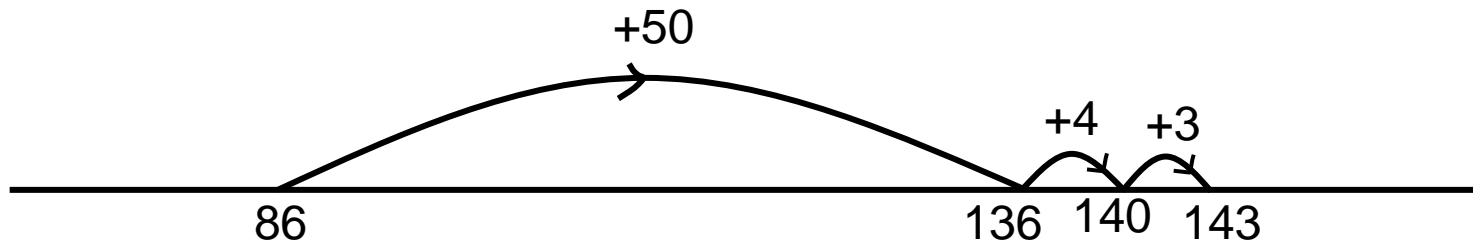


T0 + **T**0

86 + 57



Start at 86 (the larger number) on the number line. *Partition* the smaller number 57 into tens and units and count on the multiples of 10 first and then the ones.



$$86 + 57 = 143$$

ADDITION

Using a number line to add too much and then subtract (*compensate*)

$$\begin{array}{r} \text{HT0} + \text{T0} \\ 754 + 96 \end{array}$$

Why are you subtracting when you should be adding?

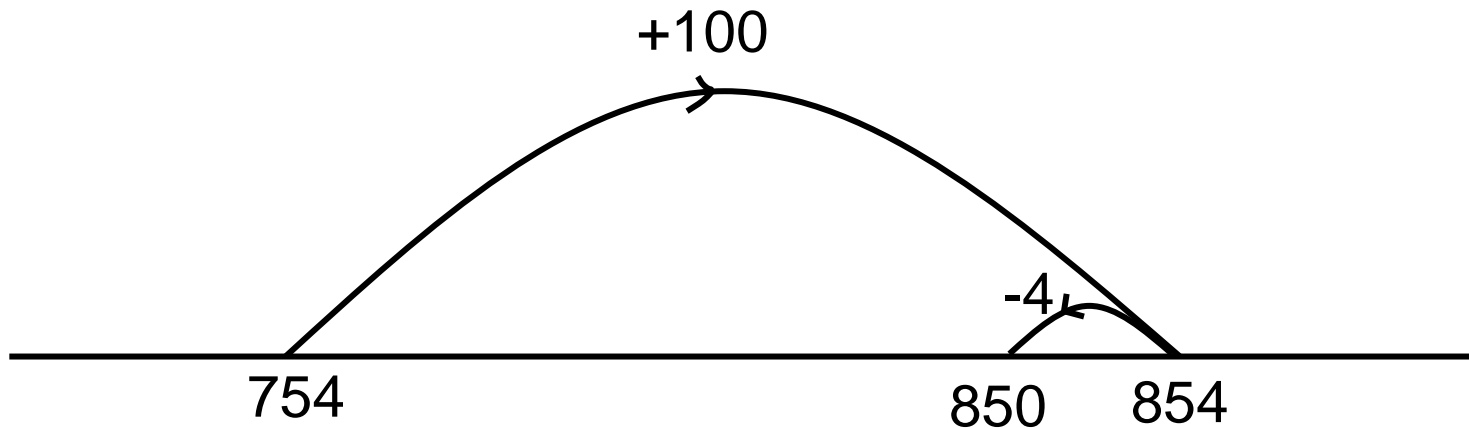
I noticed that 96 is close to 100. 100 is easier to add than 96 but that means I've added 4 too many. I need to subtract 4 from the number I reach.



$$\begin{array}{l} \text{HTO} + \text{TO} \\ 754 + 96 \end{array}$$



Start with the larger number 754. Add on 100 and then subtract 4.



$$754 + 96 = 850$$

ADDITION

$$\begin{array}{r} \text{HT0 + T0} \\ 625 + 148 \end{array}$$

Expanded method: moving on from adding the *most significant digits* first to adding *least significant digits* first

Why switch to adding the units (*least significant digits*) first?

I know that I can add numbers in any order and the total will be the same. My teacher has told me that I need to practise adding the ones first. The next method I will learn works this way. I must remember to line the numbers up in the correct columns.



HTO + HTO

$$625 + 148$$

Add *most significant digits* first:
(in this example, **hundreds**)

$$\begin{array}{r} 625 \\ + 148 \\ \hline 700 \\ 60 \\ 13 \\ \hline 773 \end{array} \quad \begin{array}{l} 600 + 100 \\ 20 + 40 \\ 5 + 8 \end{array}$$

$$\begin{array}{r} 625 \\ + 148 \\ \hline 13 \\ 60 \\ 700 \\ \hline 773 \end{array} \quad \begin{array}{l} 5 + 8 \\ 20 + 40 \\ 600 + 100 \end{array}$$

Mentally add
 $700 + 60 + 13 = 773$

$$625 + 148 = 773$$

ADDITION

Using a standard method

HTO + HTO

587 + 475

Why do you say $80 + 70$
instead of $8 + 7$?

I need to remember the value
of each digit, so I know the
size of the numbers I am
adding and whether my
answer makes sense.



HTO + HTO
587 + 475

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \text{11} \end{array}$$

$7 + 5 = 12$
Place the **2** in the ones column and carry the **10** forward to the tens column.

$80 + 70 = 150$ then $+ 10$ (carried forward) which totals **160**.
Place **60** in the tens column and carry the **100** forward to the hundreds column.

$500 + 400 = 900$ then $+ 100$ which totals **1000**. Place this in the thousands column.

$587 + 475 = 1062$

SUBTRACTION

$$\begin{array}{r} T0 - T0 \\ 84 - 56 \end{array}$$

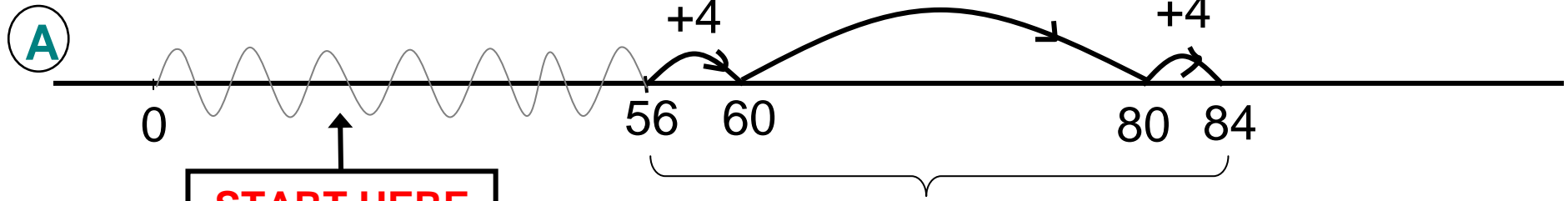
How do you decide whether to count on or count back?

A Counting on or **B** counting back?

If the numbers are close together like $203 - 198$ it's quicker to count on. If they're a long way apart like $203 - 5$ it's quicker to take away. Sometimes I count on because that's easier than taking away.



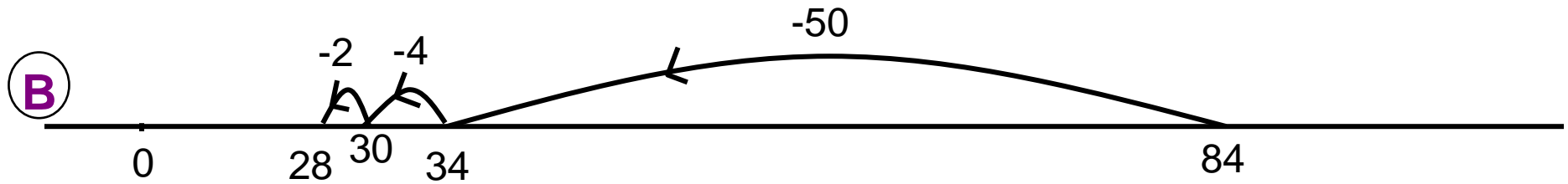
T0 - T0
84 - 56



START HERE

Start by 'taking away' (crossing out) the 56.

Find the *difference* between the two numbers.
Count on from 56 to 84.
 $20 + 4 + 4 = 28$



Partition 56 and count back (subtract) 50 and then 6.

START HERE

84 - 56 = 28

SUBTRACTION

$$\begin{array}{r} \text{HT0} - \text{HT0} \\ 954 - 586 \end{array}$$

Complementary addition

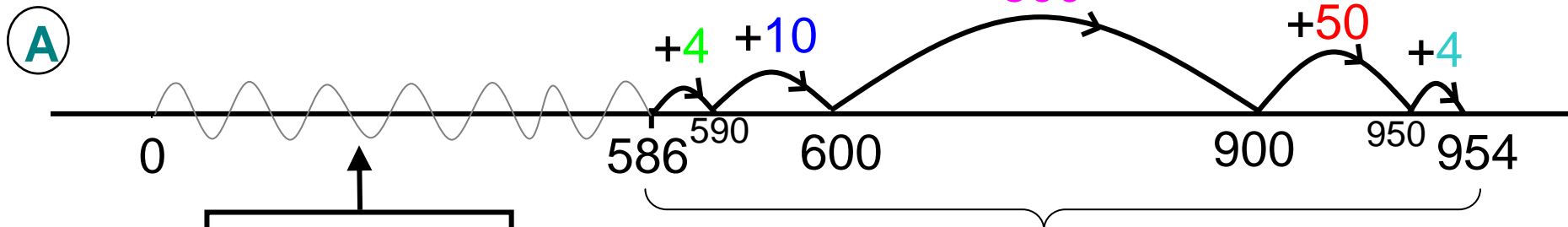
- A** Number line
- B** Written method

The number line method is very clear. Why do you use method B and write the numbers vertically?

I could make mistakes. Method B helps me line the numbers up and see what I need to add.



HTO - HTO
954 - 586



START HERE

'Take away' the 586.

Find the *difference* between the two numbers.
 Count on from 586 to 954.
 $300 + 50 + 10 + 4 + 4 = 368$

(B)

START HERE

Count on to the next multiple of 10.

Count on to the next multiple of 100.

Count on in 100s.

Count on to the larger number in the calculation which is 954.

$$\begin{array}{r}
 954 \\
 - 586 \\
 \hline
 4 \\
 10 \\
 300 \\
 50 \\
 4 \\
 \hline
 368
 \end{array}$$

4 To make 590
10 To make 600
300 To make 900
50 To make 950
4 To make 954

$954 - 586 = 368$

SUBTRACTION

Working towards a standard method (*decomposition*)

$$\begin{array}{r} \text{HTO} - \text{TO} \\ 154 - 37 \end{array}$$

Why do you need to rearrange the numbers $50 + 4$ and rewrite them as $40 + 14$?

The whole number is 154. It is possible to subtract 7 but for this method I need to do one subtraction in each column. So I exchange one ten from the tens column for ten ones in the ones column.



HT0 - T0
154 - 37

Both these numbers are partitioned into their HT0 parts, so we can do 3 easier calculations.

54 is the same value as 40 10 4.
 Now 7 can be subtracted from 14.

Subtract the units, tens, then hundreds.

$$\begin{array}{r} 100 + 50 + 4 \\ - \quad 30 + 7 \end{array}$$

$$\begin{array}{r} 100 + \cancel{50} + 4 \\ - \quad 30 + 7 \end{array}$$

$$\begin{array}{r} 100 + 40 + 14 \\ - \quad 30 + 7 \end{array}$$

$$100 + 10 + 7 = 117$$

$$100 - 0 = 100$$

$$40 - 30 = 10$$

$$14 - 7 = 7$$

Here the answers from each calculation are added to give the answer.

154 - 37 = 117

SUBTRACTION

Standard method (*decomposition*)

$$\begin{array}{r} \text{HT0} - \text{HT0} \\ 754 - 286 \end{array}$$

Why didn't you use
the standard
method straight
away?

Because all the stages I
have learnt before have
really helped me
understand exactly
what I'm doing.



HTO - HTO

754 - 286

54 is the same value as
 $\boxed{40} + \boxed{10} + \boxed{4}$.
 Now 6 can be subtracted
 from 14.

740 is the same value as
 $\boxed{600} + \boxed{100} + \boxed{40}$.
 Now 80 can be
 subtracted from 140.

Or, more efficiently
 the *standard method*.

$$\begin{array}{r}
 700 + \overset{40}{\cancel{50}} + \overset{1}{4} \\
 - 200 + 80 + 6
 \end{array}$$

$$\begin{array}{r}
 600 + \overset{1}{\cancel{700}} + 40 + 14 \\
 - 200 + 80 + 6
 \end{array}$$

$$\begin{array}{r}
 600 + 140 + 14 \\
 - 200 + 80 + 6
 \end{array}$$

$$400 + 60 + 8 = 468$$

$$\begin{array}{r}
 \overset{6}{\cancel{7}}\overset{4}{\cancel{5}}4 \\
 - 286 \\
 \hline
 468
 \end{array}$$

$$754 - 286 = 468$$

MULTIPLICATION

Introducing multiplication on a number line

$$\begin{array}{r} T0 \times 0 \\ 14 \times 5 \end{array}$$

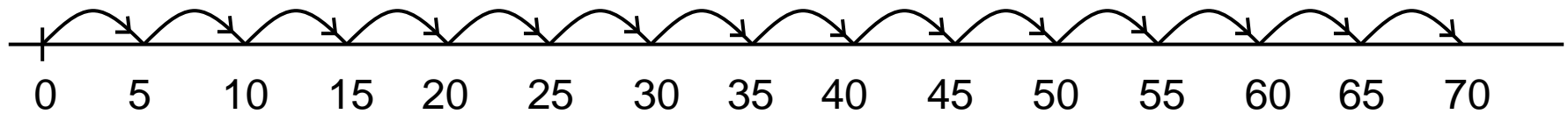
How is multiplication the same as repeated addition?

The number line helps me see each group of 5 clearly.
If I add 5 fourteen times, that is the same as 5 multiplied by 14 (5×14). I can make 14 individual jumps of 5 along the number line, or 1 jump of 5×10 and 1 jump of 5×4 . Table facts will help me do this more quickly.

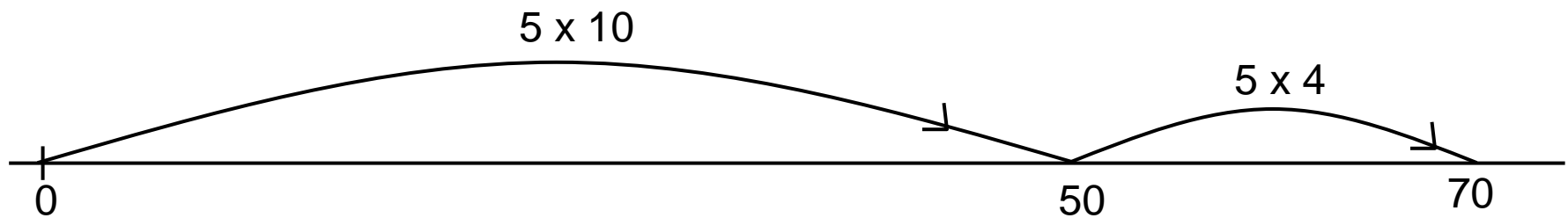


T 0 x 0
14 x 5

The number line shows 5 multiplied by 14. This is equal to 14 multiplied by 5 (14 jumps of 5 on the number line).



Multiplication is *repeated addition*.



Using table facts to make bigger jumps is more efficient.

14 x 5 = 70

GRID MULTIPLICATION

T0 X 0

14 x 5

Why do you
partition the
numbers into tens
and ones?

It doesn't take
long!
I can see what I
have to multiply
very easily.



T0 X 0
14 x 5

Partition T0 number into tens and units parts.

14 becomes **10** and **4**

	14 x 5		
X	10	4	
5	50	20	70

The *part-products* are totalled to give the *final product* or answer of 70.

50 comes from multiplying 10 by 5. It is called a *part-product*.

20 comes from multiplying 4 by 5. Another *part-product*.

14 x 5 = 70

GRID MULTIPLICATION

TO X TO
46 x 32

Isn't it difficult to multiply 40 by 30?

I know that 30 is 3×10 and multiplying by 10 is easy so I do $40 \times 3 \times 10 = 120 \times 10 = 1200$.

You've got to do a lot of calculations – don't you get confused?



The layout of the grid helps me organise what I have to do. I like this method.

T O X T O
46 x 32

Both numbers are *partitioned* into their tens and ones parts,

46 becomes **40** and **6** and **32** becomes **30** and **2**.

46 x 32

X	40	6	
30	1200	180	1380
2	80	12	92
			1472

The *part products* are added in stages to give the final *product* or answer of 1472.

46 x 32 = 1472

MULTIPLICATION

Grid method, **Expanded method**
and **Compact method**

T0 X 0

23 x 8

What are the brackets for in the expanded method?

They remind me which numbers I am multiplying.
I also have to remember to line the numbers up as hundreds, tens and ones.

Why do you multiply 3 by 8 first in the compact method?
In all the other methods I've noticed that you've multiplied the tens number first!



I multiply the units first so I can carry forward any tens I need to!
This method is very quick but I have to remember to add on any numbers I carry forward.

T O X O

$$23 \times 8$$

GRID METHOD

X	20	3	
8	160	24	184

20 multiplied by 8 equals 160.
3 multiplied by 8 equals 24.

Final product from totalling the *part-products*.

COMPACT METHOD (short multiplication)

$$\begin{array}{r} \text{HTO} \\ 23 \\ \times 8 \\ \hline 184 \\ \hline 2 \end{array}$$

3 multiplied by 8 equals 24 (the first *part product*).

2 is the 2 tens that need to be carried forward and added to the next *part product*.

20 multiplied by 8 equals 160 (2nd *part product*), plus the 2 tens equals 180.

The digits are put in the correct columns, to give the answer 184.

$$23 \times 8 = 184$$

MULTIPLICATION

Grid method, **Expanded method**
and **Compact method**

T0 X T0
46 x 32

I recognise the long multiplication method. How do you multiply 46 by 30?

Well!... I know that 46×30 is the same as $46 \times 3 \times 10$. I know my answer will end in zero when I multiply this whole number by 10. So... I put the zero in first. Then I multiply 46×3 using the short multiplication method.



TO X TO

46 x 32

GRID METHOD

X	40	6	
30	1200	180	1380
2	80	12	92
			1472

COMPACT METHOD

(long multiplication)

$$\begin{array}{r}
 46 \\
 \times 32 \\
 \hline
 92 \\
 1380 \\
 \hline
 1472
 \end{array}$$

(46 x 30)

(46 x 2)

46 x 30 is the same as 46 x 3 x 10.

46 x 2 mentally or by short multiplication.

EXPANDED METHOD

The 4 *part products* are set out vertically underneath the calculation.

Part products totalled to give final product.

$$\begin{array}{r}
 46 \\
 \times 32 \\
 \hline
 1200 \quad (40 \times 30) \\
 180 \quad (6 \times 30) \\
 80 \quad (40 \times 2) \\
 12 \quad (6 \times 2) \\
 \hline
 1472
 \end{array}$$

$$46 \times 32 = 1472$$

DIVISION

$$T0 \div 0$$

$$29 \div 3$$

Introducing division on a number line

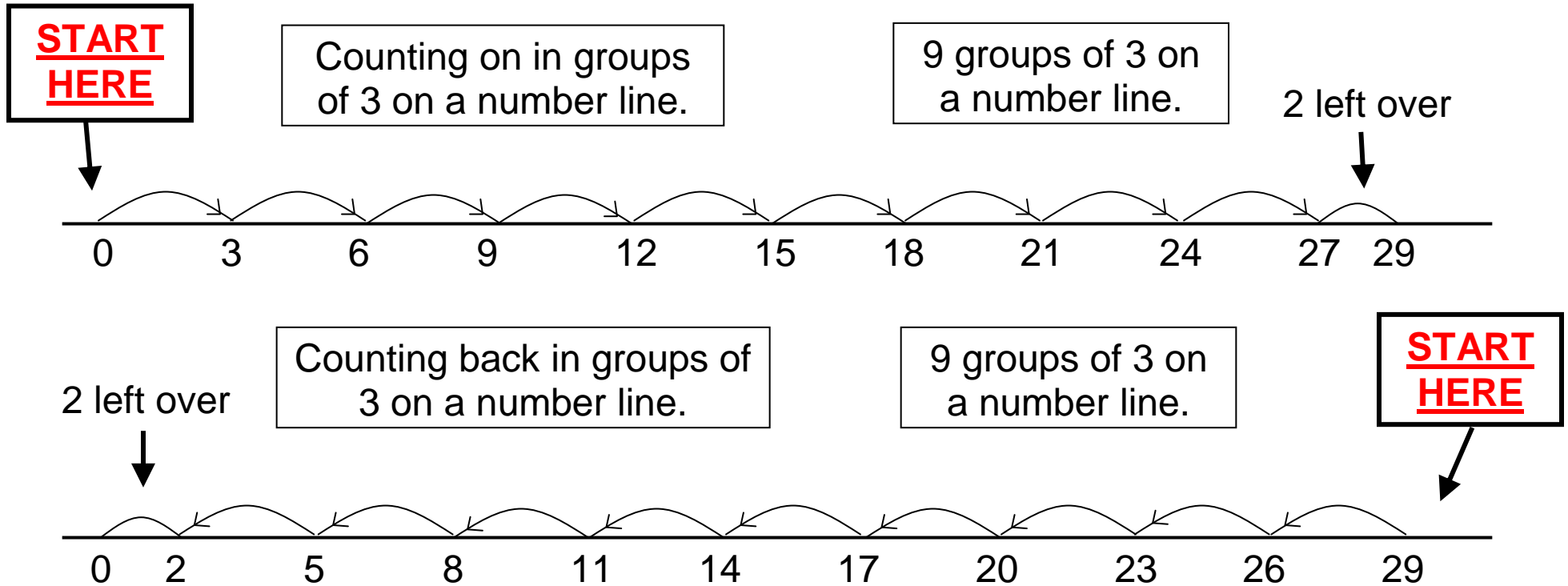
Why are you adding on one line and subtracting on the other? And what has subtraction got to do with division?

I need to see how many groups of 3 there are in 29, so I either add on or take away groups of 3 until I can't add or take any more. Using the subtraction method will help me later on.



$$T0 \div 0$$

$$29 \div 3$$



There are 9 groups of 3 in 29, with 2 left over.

$$29 \div 3 = 9 \text{ r}2$$

DIVISION

$T0 \div 0$

$72 \div 5$

Chunking on a number line

I've never heard of chunking before!
How does this help with division?

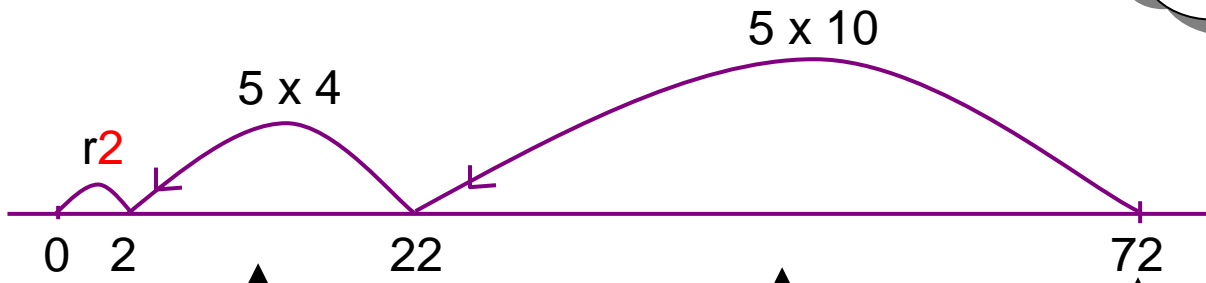
If I can, I try to take out 10 groups of the number I'm dividing by. This is a big chunk and makes the calculation easier. But I can take out chunks that are any number of groups.



$$70 \div 0$$

$$72 \div 5$$

Numberlines can be **vertical** or **horizontal**.



Subtract 4 groups of 5 (20) from 22 to land on 2.

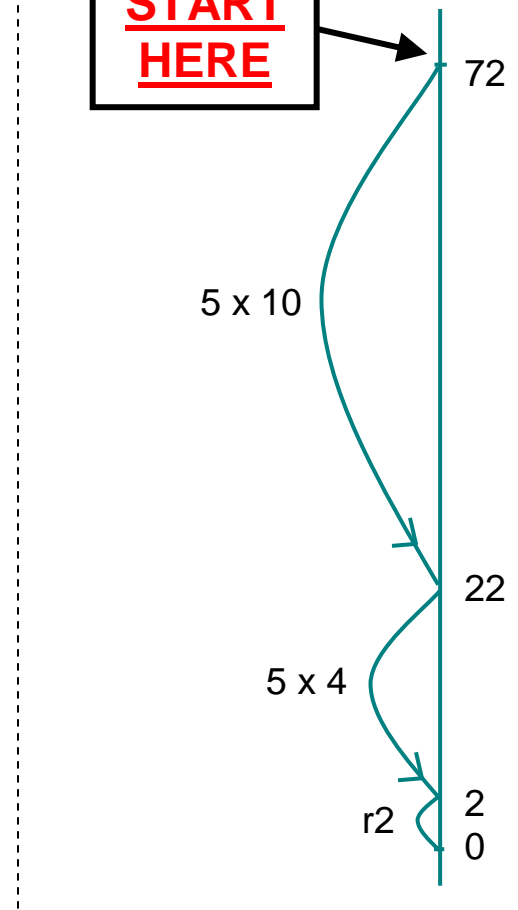
Subtract 10 groups of 5 (50) from 72 to land on 22.

START
HERE

14 groups of 5 subtracted altogether.

2 left!
This is the *remainder*.

START
HERE



$$72 \div 5 = 14 \text{ r}2$$

DIVISION BY CHUNKING

HTO \div 0

256 \div 7

How do you decide
what size chunk to
subtract?

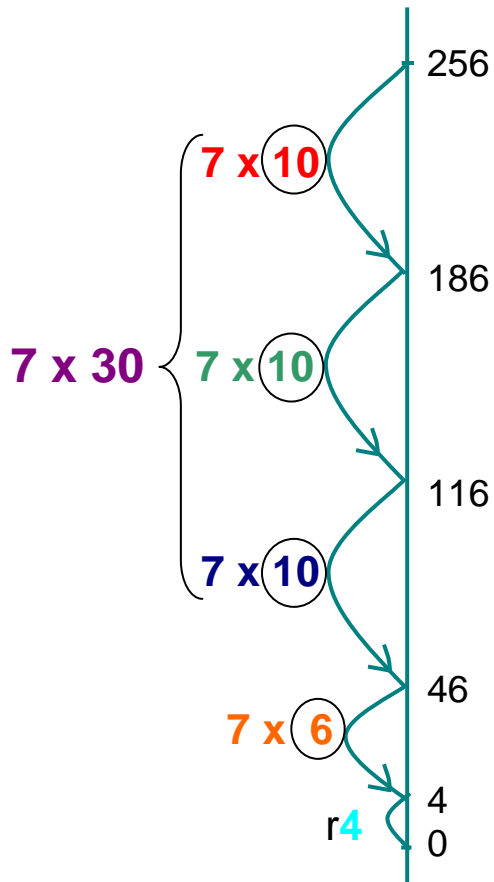
I look for chunks of 10
first. If I take bigger
chunks it makes the
calculation quicker
and easier. Method **C**
is shorter and more
efficient than **B**.



$$HTO \div 0$$

$$256 \div 7$$

(A)



How many groups of 7 in 256?

(B)

$$\begin{array}{r} 256 \\ -70 \text{ (7 x 10)} \\ \hline 186 \\ -70 \text{ (7 x 10)} \\ \hline 116 \\ -70 \text{ (7 x 10)} \\ \hline 46 \\ -42 \text{ (7 x 6)} \\ \hline 4 \end{array}$$

Subtract chunks of 70 (7 x 10).

How many groups of 7 in 46?

Total the numbers of groups of 7.

$$10 + 10 + 10 + 6 = 36$$

(C)

$$\begin{array}{r} 256 \\ -210 \text{ (7 x 30)} \\ \hline 46 \\ -42 \text{ (7 x 6)} \\ \hline 4 \end{array}$$

Subtract one large chunk of 210 (7 x 30).

36 groups of 7 have been subtracted and there is 4 left over.

$$256 \div 7 = 36 \text{ r}4$$

SHORT COMPACT DIVISION

Isn't it easier to say
'how many 3s in 4?'

I need to remember the value
of each digit so I know
whether my answer makes
sense. I will only use this
method when I am confident
with mental and chunking
methods of division.



HTO ÷ O

471 ÷ 3

$$\begin{array}{r} 1 \\ 3 \overline{) 471} \end{array}$$

Q: What is the largest number of hundreds that will divide exactly by 3?

A: 300 divided by 3 = 100. This leaves 100 which is exchanged for ten tens in the tens column.

$$\begin{array}{r} 15 \\ 3 \overline{) 471} \end{array}$$

Q: What is the largest number of tens that will divide exactly by 3?

A: 150 divided by 3 = 50. This leaves 20 which is exchanged for 20 ones in the ones column.

$$\begin{array}{r} 157 \\ 3 \overline{) 471} \end{array}$$

Q: What is the largest number of ones that will divide exactly by 3?

A: 21 divided by 3 = 7

471 ÷ 3 = 157

CALCULATIONS IN CONTEXT

All the methods in this booklet support children in using their mental and written skills to solve calculations. Children need to be encouraged to use the method that they understand and can use confidently.

It is important that children are able to choose the most appropriate method for the calculation. For example:

$$4003 - 3998$$

These numbers are very close together and so counting up on a number line (actual or imagined) would be the most efficient method.

$$200 \div 4$$

Dividing by 4 is the same as halving and halving again. As it is easy to halve 200 and easy to halve 100, this would be the most efficient method.

Using and applying appropriate skills is very important, when calculations are needed to solve a problem.

4 C.DS at £2.99 – how much altogether?

£2.99 is almost £3.00 and so round up, multiply, then adjust:

$$4 \times £3.00 = £12.00$$

$$£12.00 - 4p = £11.96$$

Improving your own skills

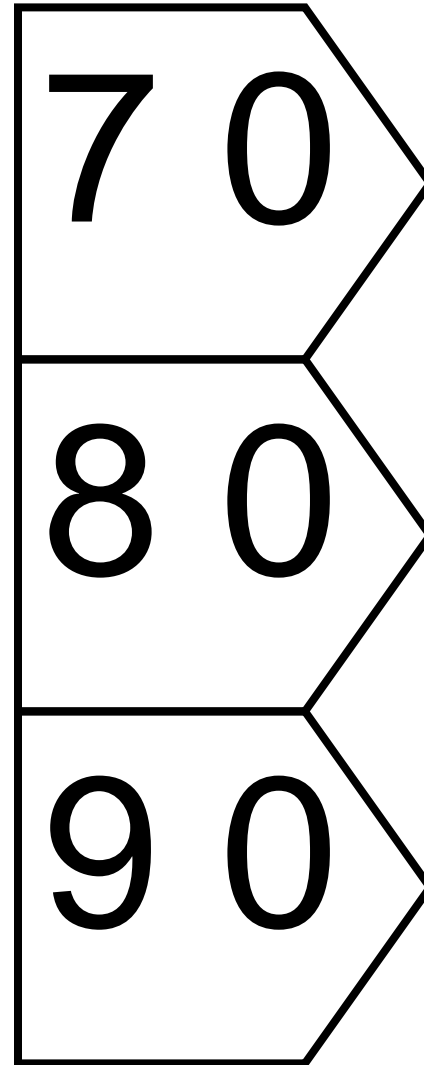
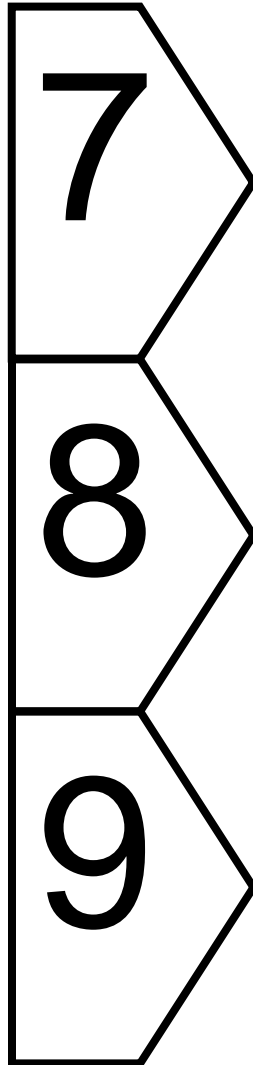
Many adults think that they aren't very good at Maths. If you think it's time that you did something about your own Maths, there are lots of sources of help.

- There are national telephone and internet helplines available to help find an appropriate course.

Learndirect: www.learndirect.co.uk 0800 100 900
Basic SkillsAgency: www.basic-skills.co.uk/learners 0800 700 987
BBC Skillswise: www.bbc.co.uk/skillswise

- There are several websites designed to help students of all ages find out about different topics in Maths:
 - The BBC site (www.bbc.co.uk) has excellent sections for revision at KS2 and KS3 (www.bbc.co.uk/revisewise), and the GCSE and Skillswise sections also give worked examples of mathematical problems - particularly useful when your child doesn't understand her homework and you don't either.....
 - The DfES0 site for parents (www.parentcentre.gov.uk) is the best source of information about teaching in schools, and how to support your child's learning at home.
 - The Parents Online site (www.parentsonline.gov.uk) gives information about children's education, and how parents can support children's education - particularly using the Internet.

Place Value Cards



1 0 0 0
2 0 0 0
3 0 0 0
4 0 0 0

1 0 0
2 0 0
3 0 0
4 0 0

5 0 0 0

6 0 0 0

7 0 0 0

8 0 0 0

5 0 0

6 0 0

7 0 0

8 0 0

