Maths - Teaching for Mastery

Greater Depth Policy

In God's love, aspire and achieve to be the best'
1 Corinthians 16:14 'Do everything in love.'

WINWICK CE PRIMARY SCHOOL AIMS/ETHOS

At Winwick CE Primary School we follow the National Curriculum for mathematics, which is statutory for all maintained schools, it aims to ensure that all pupils:

• become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems

• reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justifications or proof using mathematical language

• can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of smaller steps and persevering in seeking solutions.

What is Mastery?

Supplementary subject-specific guidance for mathematics produced by Ofsted (2012)

A description of outstanding learning in mathematics includes a statement that pupils;

"think for themselves, and are prepared to persevere when faced with challenges, showing a confidence that they will succeed. They embrace the value of learning from mistakes and false starts."

Mastery is a deep and secure learning for all. It is where learning is broken down into discrete units and presented in a logical order. Pupils are required to demonstrate mastery of learning from each unit before moving onto the next. Mastery is achieved when children can skilfully apply their learning in a more complex and in-depth situation.

What is the difference between Mastery and Mastery at Greater Depth?

Mastery

- To be able to use mathematical concepts, facts and procedures appropriately, flexibly and fluently.

- To use reasoning both to explain known mathematical concepts and procedures and to use them to solve problems.

Mastery with greater depth

- To solve non-routine problems (i.e. where the approach is not immediately obvious) demonstrating creativity and imagination.
- To explore and investigate mathematical contexts and structures, to communicate results clearly and systematically and to explain, generalise and prove results.

What does Greater Depth look like in the classroom?

- All children are entitled to reasoning and deepening to stretch their understanding.
- Different students may be working in greater depth in different concepts.
- There should be reasoning throughout the lesson, this reasoning will pave the way for students to be able to access and do the deepening activity.
- Deepening should not be taught in isolation.

How is Greater Depth achieved in the classroom?

1) Fluency

Efficiency implies that the student does not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub problems and making use of intermediate results to solve the problem.
Accuracy depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.
Flexibility requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the problem at hand, and also to use one method to solve a problem and another method to double-check the results.
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Fluency rests on a well-built mathematical foundation with three parts:

(1) an understanding of the meaning of the operations and their relationships to each other -- for example, the inverse relationship between multiplication and division;
(2) the knowledge of a large repertoire of number relationships, including the addition and multiplication "facts" as well as other relationships, such as how 4 × 5 is related to 4 × 50; and

(3) a thorough understanding of the base ten number system, how numbers are structured in this system, and how the place value system of numbers behaves in different operations -- for example, that 24 + 10 = 34 or 24 × 10 = 240.

- An appreciation of number and number operations, which enables mental calculations and written procedures to be performed efficiently, fluently and accurately.
- Requires good understanding of the key principles that underpin arithmetic procedures.
- Fluency – accurate, efficient, flexible.
- Learners can make decisions based on the numbers involved.
- Quick recall of key number facts.
- Fluent computational skills are dependent on accurate and rapid recall of basic number bonds within 20 and times-tables facts.
- Spending a short time every day on the basic facts quickly leads to improved fluency.

2) Reasoning
ALL children are entitled to reasoning in a lesson. Reasoning helps children understand the concept they're learning helps them apply it to other questions and helps them stretch their learning further. This can be done through: Call and Response/Recall using Maths vocabulary, showing students different reasoning questions and having them discuss, going back to the question: always sometimes never.

Please see appendix 1 for Reasoning Toolbox.

NRICH Steps in Reasoning.

1. **Describing:** simply tells what they did.
2. **Explaining:** offers some reasons for what they did which may or may not be correct and might not hang together coherently.
3. **Convincing:** confident that their chain of reasoning is right and may use words such as, 'I reckon' or 'without doubt'. The underlying mathematical argument may or may not be accurate yet is likely to have more coherence and completeness than the explaining stage. This is called inductive reasoning.
4. **Justifying:** a correct logical argument that has a complete chain of reasoning to it and uses words such as 'because', 'therefore', 'and so', 'that leads to' ...
5. **Proving:** a watertight argument that is mathematically sound, often based on generalisations and underlying structure. This is also called deductive reasoning.

Reasoning with Depth.

- **Deepening activities are ones that stretch a student's learning further and allows them to apply their understanding to solve unfamiliar problems.**

Table 5: Depth of understanding

Strand	Rating scale			
	(low) 1	2	3	4 (high)
Depth of understanding	recall of facts or application of procedures	use facts and procedures to solve simple problems	use facts and procedures to solve more complex problems	understand and use facts and procedures creatively to solve complex or unfamiliar problems

3) Problem Solving

There is an important relationship between Problem Solving and Mathematical Thinking. We cannot directly teach problem solving; however, we can support children in becoming better problem solvers through:

- Developing deep understanding of maths, in particular relationships
- Developing logical reasoning – particularly through procedure variation
- Developing knowledge of mathematical structures through conceptual Variation
- Developing analytical skills e.g. true/false question
- Providing unfamiliar contexts - Dong Nao Jin questions

Gu identified and illustrated the two forms of variation, namely “conceptual variation” and “procedural variation.” Conceptual variation aims at providing students with multiple perspectives and experiences of mathematical concepts. Procedural variation aims to provide a process for the formation of concepts step-by-step so that students' experiences in solving problems are manifested by the richness of varying problems and the variety of transferring strategies (Gu et al., 2004).

See appendix 2 for a range of problem solving heuristics

INEFFECTIVE

There is a lack of genuine problem solving tasks in teaching. Tasks tend to be routine and can be completed using a procedure that pupils know well.

Teachers lack knowledge and understanding of problem solving strategies. They do not feature in their teaching.

Teachers do not consciously vary the structure and context to problems.

Teachers rarely encourage pupils to use representations and manipulatives to represent problems mathematically.

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IMPROVING

Some teachers select non-routine problems, but other teachers do not. Teachers sometimes do not feel confident enough to work on genuine, non-routine problem solving.

Teachers effectively model a range of problem-solving strategies. However, they do not effectively support pupils to self-regulate their use of strategies.

Teachers do pay attention to context and structure when setting problems, but this is not systematic and does not support improved understanding.

Some teachers encourage pupils to use representations and manipulatives to represent problems mathematically. However, this practice is not consistently adopted by teachers throughout the school.

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EXEMPLARY

Most teachers confidently select genuine, non-routine problem-solving tasks.

Teachers know a range of strategies, which they can model effectively for pupils. They teach pupils to carefully and consciously choose the most appropriate strategy for the problem at hand.

Teaching is organised so that problems with similar structures and different contexts are presented together, and, likewise, that problems with the same context but different structures are presented together. Pupils are taught to identify similar mathematics that underlies different situations, and identify and interrogate multiple relationships between variables in one situation.

Teachers encourage pupils to use representations and manipulatives to represent problems mathematically.

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INEFFECTIVE

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Worked examples rarely feature in teaching.

Teachers rarely use the following approaches to improve pupils' use of strategies:

- Encouraging pupils to share and discuss strategies
- Encouraging pupils to interrogate and use their mathematical knowledge to solve problems
- Encouraging pupils to communicate their reasoning about their choice of strategies
- Requiring pupils to compare and evaluate multiple strategies

IMPROVING

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Teachers deploy worked examples, but they are mainly used to consider steps in a procedure and are rarely used to examine problem-solving strategies.

Teachers are confident using only some of these approaches, or they could improve in some areas.

EXEMPLARY

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Teachers use worked examples to enable pupils to examine the use of different problem solving strategies.

Teachers are confident and capable when using all of these approaches to improving pupils' use of strategies.

4) Questioning

Effective questioning is paramount to building depth in children's understanding. In order to develop higher order thinking skills, teachers at Grosvenor Road use the follow types of questions (verbally and written) to promote mastery for greater depth.

1. Create examples/special cases	**4. Modifying and changing**
Show me an example of a square number.	How can you change this time into 24-hour clock time?
2. Evaluating and correcting	**5. Generalising & conjecturing**
What is wrong with the statement and how can you correct it: When you multiply by 10 you add a '0'.	Is this always, sometimes (when?) or never true? If you add one to the top and bottom of a fraction, it gets bigger in value.
3. Comparing and organising	**6. Explaining and justifying**
What is the same and what is different about these objects: square, trapezium, and parallelogram?	Convince me that the area of the triangle is half the area of the rectangle.

1. Are you sure? How do you know?

To be able to answer 'Are you sure?' confidently, children will have needed to have checked their calculation/explanation themselves. Asking 'How do you know?' ensures this by asking them to share their reasoning with an adult, a group or the whole class. The result of this increased metacognition is children will be more reliant on themselves (and each other) for checking their understanding, method and answer.

2. What do you notice?

Asking pupils 'What do you notice?' when showing two calculations or problems at the same time can help children to see what is similar and different. It's a key question when building procedural variation. It also uncovers where a pupil's understanding is, showing December 2017 whether their current line of thinking is relevant to the current learning, in turn enabling the teacher to guide them towards more relevant thinking where necessary.

3. What's the same and what's different?

A question that serves the same purpose as ‘What do you notice?’ but is more confining in its responses, is ‘What’s the same and what’s different about these 2 calculations/problems/statements?’

4. Can you convince me?

This is another question that can help develop generalisation. Asking individuals or small groups to work together to convince you of something develops their depth of understanding and ability to reason. Here are some examples: 'Convince me that subtraction is the opposite of addition’ 'Convince me that all multiples of 8 are multiples of 2’

Is there another way?

‘Is there another way to find 25% of £80?’ ‘Is there another way to work out 47+28?’ ‘Is there another way you could have used to find all the possibilities?’ This highlights to children that in Mathematics, as in life, there can be a variety of ways to solve problems. It can also be used as a general challenge activity for pupils.

5. Is it always, sometimes or never true?

This is a great question to further develop children’s ability to generalise and, dependent on the question, their number sense. As with all the questions, it develops reasoning skills and can deepen understanding.

Summary

Greater depth mathematicians <u>should</u> be expected to take greater responsibility for their learning by editing their own work in the same way they would in literacy.

Children who are working at greater depth <u>would</u> confidently and independently:

- access mathematical problems presented in a wide range of different, complex ways
- are able to justify and prove their conjectures when reasoning;
- ask their own mathematical questions and follow their own lines of enquiry when exploring an open-ended mathematical problem.

Provision for children working at greater depth <u>must</u>:

- model higher-level reasoning skills (justification and proving) and encourage children to use them;
- model mathematical questioning during open-ended mathematical problems and encourage children to ask them;
- provide complex mathematical problems (open and closed) with a variety of contexts and support children initially to access these, until they can do them independently;
- motivate children to be confident and resilient enough to do the above.

Appendix 1- Reasoning handbook (see separate document)
Appendix 2- Problem solving heuristics (see separate document)